fourth GRADE

2018-2019 Curriculum Guide

September 10– October 17

<u>Eureka</u>

Module 1: *Place Value, Rounding, and Algorithms for Addition and Subtraction*

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

Table of Contents

I.	Module Performance Overview	p. 3
II.	Lesson Pacing Guide	p. 4-5
III.	Unit 2 NJSLS Unpacked Math Standards	p. 6-11
IV.	Assessment Framework	p. 12
V.	Ideal Math Block	p. 13
VI.	Eureka Lesson Structure	p. 14
VII.	PARCC Evidence Statements	p. 15
VIII.	Number Talks	p. 16
IX.	Student Friendly Rubric	p. 17
X.	Mathematical Representations	p. 18-20
XI.	Mathematical Discourse/ Questioning	p. 21-25
XII.	Conceptual & Procedural Fluency	p. 26
XIII.	Evidence of Student Thinking	p. 27
XIV.	Effective Mathematical/ Teaching Practices	p. 28-30
XV.	5 Practices for Orchestrating Productive Mathematics Discourse	p. 31
XVI.	Math Workstations	p. 32-34
XVII.	Data Driven Instruction/ Math Portfolios	p. 35-37
XVIII.	Authentic Performance Assessment	p. 38-39
XIX.	Additional Resources	p. 40

Module 1 Performance Overview

- In this module, students extend their work with whole numbers. They begin with large numbers using familiar units (hundreds and thousands) and develop their understanding of millions by building knowledge of the pattern of times ten in the base ten system on the place value chart. They recognize that each sequence of three digits is read as hundreds, tens, and ones followed by the naming of the corresponding base thousand unit (thousand, million, billion).
- In Topic A, students build the place value chart to 1 million and learn the relationship between each place value as 10 times the value of the place to the right. Students manipulate numbers to see this relationship, such as 30 hundreds composed as 3 thousands. They decompose numbers to see that 7 thousands is the same as 70 hundreds. As students build the place value chart into thousands and up to 1 million, the sequence of three digits is emphasized. They become familiar with the base thousand unit names up to 1 billion. Students fluently write numbers in multiple formats: as digits, in unit form, as words, and in expanded form up to 1 million.
- In Topic B, students use place value to compare whole numbers. Moving away from dependency on models and toward fluency with numbers, students compare numbers by observing across the entire number and noticing value differences. For example, in comparing 12,566 to 19,534, it is evident 19 thousands is greater than 12 thousands because of the value of the digits in the thousands unit. Additionally, students continue with number fluency by finding what is 1, 10, or 100 thousand more or less than a given number.
- In Topic C, students round to any place using the vertical number line and approximation. The vertical number line allows students to line up place values of the numbers they are comparing. Students learn the advantages to rounding to any place value, which increases accuracy. To round 34,108 to the nearest thousand, students find the nearest multiple, 34,000 or 35,000, by seeing if 34,108 is more than or less than halfway between the multiples.
- Moving away from special strategies for addition, students develop fluency with the standard addition. They practice and apply the algorithm within the context of word problems and assess the reasonableness of their answers using rounding.
- Following the introduction of the standard algorithm for addition in Topic D, the standard algorithm for subtraction replaces special strategies for subtraction in Topic E. Students practice decomposing larger units into smaller units. As in Grades 2 and 3, students continue to decompose all necessary digits before performing the algorithm, allowing subtraction from left to right, or, as taught in the lessons, from right to left. Students use the algorithm to subtract numbers from 1 million allowing for multiple decompositions.
- Topic F includes multi-step addition and subtraction word problems. Students also continue using a variable to represent an unknown quantity. To culminate the module, students are given tape diagrams or equations and are encouraged to use creativity and the mathematics learned during this module to write their own word problems to solve using place value understanding and the algorithms for addition and subtraction.

Module 1: *Place Value, Rounding, and Algorithms for Addition and Subtraction*

Pacing:
September 10- October 17
24 Days

Topic	**Lesson**	**Lesson Objective/ Supportive Videos**
Topic A: Place Value of Multi-Digit Whole Numbers	Lesson 1	Interpret a multiplication equation as a comparison. https://www.youtube.com/watch?v
	Lesson 2	Recognize a digit represents 10 times the value of what it represents in the place to its right. https://www.youtube.com/watch?v
	Lesson 3	Name numbers within 1 million by building understanding of the place value chart and placement of commas for naming base thousand units. https://www.youtube.com/watch?v
	Lesson 4	Read and write multi-digit numbers using base ten numerals, number names, and expanded form. https://www.youtube.com/watch?v
Topic B: Comparing Multi-Digit Whole Numbers	Lesson 5	Compare numbers based on meanings of the digits, using >, <, or = to record the comparison. https://www.youtube.com/watch?v
	Lesson 6	Find 1, 10, and 100 thousand more and less than a given number. https://www.youtube.com/watch?v
Topic C: Rounding Multi-Digit Whole Numbers	Lesson 7	Round multi-digit numbers to the thousands place using the vertical number line. https://www.youtube.com/watch?v
	Lesson 8	Round multi-digit numbers to any place using the vertical number line. https://www.youtube.com/watch?v
	Lesson 9	Use place value understanding to round multi-digit numbers to any place value. https://www.youtube.com/watch?v
	Lesson 10	Use place value understanding to round multi-digit numbers to any place value using real world applications. https://www.youtube.com/watch?v

Mid Module Assessment **September 25-26, 2018**		
Topic D: Multi-Digit Whole Number Addition	Lesson 11	Use place value understanding to fluently add multi-digit whole numbers using the standard addition algorithm and apply the algorithm to solve word problems using tape diagrams. https://www.youtube.com/watch?v
	Lesson 12	Solve multi-step word problems using the standard addition algorithm modeled with tape diagrams and assess the reasonableness of answers using rounding. https://www.youtube.com/watch?v
Topic E: Problem Solving with Perimeter and Area	Lesson 13	Use place value understanding to decompose to smaller units once using the standard subtraction algorithm and apply the algorithm to solve word problems using tape diagrams. https://www.youtube.com/watch?v
	Lesson 14	Use place value understanding to decompose to smaller units up to 3 times using the standard subtraction algorithm, and apply the algorithm to solve word problems using tape diagrams. https://www.youtube.com/watch?v
	Lesson 15	Use place value understanding to fluently decompose to smaller units multiple times in any place using the standard subtraction algorithm, and apply the algorithm to solve word problems using tape diagrams. https://www.youtube.com/watch?v
	Lesson 16	Solve two-step word problems using the standard subtraction algorithm fluently modeled with tape diagrams and assess the reasonableness of answers using rounding. https://www.youtube.com/watch?v
Topic F: Addition and Subtraction Word Problems	Lesson 18	Solve multi-step word problems modeled with tape diagrams and assess the reasonableness of answers using rounding. https://www.youtube.com/watch?v
	Lesson 19	Create and solve multi-step word problems from given tape diagrams and equations. https://www.youtube.com/watch?v
End Of Module Assessment **October 15-17, 2018**		

NJSLS Standards:

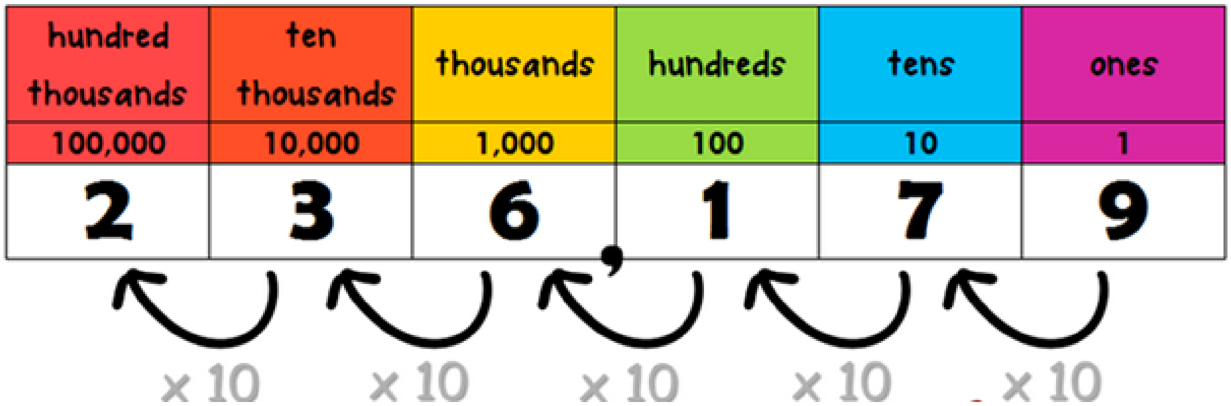
4.OA.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

- Use graphic organizers to help identify unknowns to create equations and solve a word problem based on clues in the word problem. Some operations can be used interchangeably to create different equations that solve the same word problem.
- Variables can be used to represent an unknown in any part of an equation.
- Emphasize the proper use of the equal (=) sign and the improper use (3+7=10-5=5) Relate the terms in the equation to the context in the word problem.
- Students should be able to identify the unknown(s) in the problem and use a variable to represent that term in context.
- Solve multi-step word problems involving any of the four operations. Explain why an answer is reasonable, such as using mental computation and estimation strategies.

4.NBT.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*

- Gain a conceptual understanding of decomposing numbers which later lead to computation of addition, subtraction, multiplication, and division.
- Understand the place value structure of the base-ten number system. Identify patterns when dividing or multiplying by 10.
- The value of a number is determined by the place of its digits.
- This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10.
- In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with.
- In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. Because of this, multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left. A quantitative relationship exists between the

digits in place value positions of a multi-digit number.



4.NBT.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

- Become aware of the greatest place value in a number.
- Develop a clear understanding of the value of the digits based on where they are placed in a number.
- Eliminate misconception of writing numbers as you hear them, such as writing one thousand two as 10002 by using place value boxes and grid paper.
- Write and compare numbers in three forms: expanded, word (number names), and standard (base ten numerals). Students should be able to compare numbers represented in different ways.

Example:
Compare a number written in expanded form to a number written in number names (900,000 +11 □ nineteen thousand, nine hundred ninety one)

- Compare two multi-digit numbers using >, =, and < symbols.
- Whole numbers are read from left to right using the name of the period. Comparison symbols can be used to show relationship between number values. The placement of a digit dictates its value, how it is read, written, and compared.
- This standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is 285 = 200 + 80 + 5. Written form or number name is two hundred eighty-five.However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones.
- To read numerals between 1,000 and 1,000,000, students need to understand the role of commas. Each sequence of three digits made by commas is read as hundreds, tens, and

ones, followed by the name of the appropriate base-thousand unit (thousand, million, billion, trillion, etc.). Thus, 457,000 is read “four hundred fifty seven thousand.” The same methods students used for comparing and rounding numbers in previous grades apply to these numbers, because of the uniformity of the base-ten system.

4.NBT.3	Use place value understanding to round multi-digit whole numbers to any place.

- Use a number line and/ or hundreds chart to determine what base ten numbers the number being rounded is closest to. Use anchoring and visualization techniques, such as place value charts.
- Create rounding rules and utilize number sense to emphasize when the rounded digit rounds up or stays the same. Determine possible range for answer.
- Round multi-digit whole numbers, explain the process, and apply to real life situations. Rounding numbers should result in using number sense and estimation not just following a rule. This supports students in developing a rule on their own.
- When students are asked to round large numbers, they first need to identify which digit is in the appropriate place.

Example:
Round 76,398 to the nearest 1000.
• Step 1: Since I need to round to the nearest 1000, then the answer is either 76,000 or 77,000.
• Step 2: I know that the halfway point between these two numbers is 76,500.
• Step 3: I see that 76,398 is between 76,000 and 76,500.
• Step 4: Therefore, the rounded number would be 76,000.

4.NBT.4	Fluently add and subtract multi-digit whole numbers using the standard algorithm.[Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.]

- Recognize the need of regrouping and not just subtracting the smaller digit from the larger one.
- Fluency refers to accuracy, efficiency (using a reasonable amount of steps and times) and flexibility (variety of strategies learned previously if needed).
- Computation involves taking apart and combining numbers using a variety of approaches. Flexible methods of computation involve grouping numbers in strategic ways: partial sum, regrouping, and trade first.
- Addition and subtraction algorithms are abbreviations or summaries of the connection between math drawings and written numerical work.

Computation algorithm

A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. In mathematics, an algorithm is defined by its steps and not by the way those steps are recorded in writing. With this in mind, minor variations in methods of recording standard algorithms are acceptable. Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm.

Computation strategy
Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

- **This is the first grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use.**

- As with addition and subtraction, students should use methods they understand and can explain. Visual representations such as area and array diagrams that students draw and connect to equations and other written numerical work are useful for this purpose. By reasoning repeatedly about the connection between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or summaries of their reasoning about quantities.

- Students can invent and use fast special strategies while also working towards understanding general methods and the standard algorithm. One component of understanding general methods for multiplication is understanding how to compute products of one-digit numbers and multiples of 10, 100, and 1000. This extends work in Grade 3 on products of one-digit numbers and multiples of 10. We can calculate 6 x 700 by calculating 6 x 7 and then shifting the result to the left two places (by placing two zeros at the end to show that these are hundreds) because 6 groups of 7 hundred is 6 x 7 hundreds, which is 42 hundreds, or 4,200. Students can use this place value reasoning, which can also be supported with diagrams of arrays or areas, as they develop and practice using the patterns in relationships among products such as 6 x 7, 6 x 70, 6 x 700, and 6 x 7000. Products of 5 and even numbers, such as 5 x 4, 5 x 40, 5 x400, 5 x 4000 and 4 x 5, 4 x 50, 4 x 500, 4 x 5000 might be discussed and practiced separately afterwards because they may seem at first to violate the patterns by having an “extra” 0 that comes from the one-digit product.

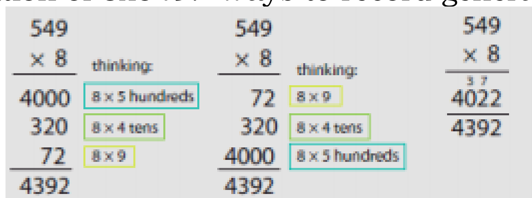
Computation of 8x549 connected to an area model

	549=500	+40	+9
8	8 X 500= 8 X 5 hundreds= 40 hundreds	8 X 40 8 X 4 tens= 32 tens	8 X 9 =72

Each part of the region above corresponds t one of the terms in the computation below.
8x549=8x(500+40+9) =8x500+8x40+8x9

This can also be viewed as finding how many objects are in 8 groups of 549 objects by finding the cardinalities of 8 groups of 500, 8 groups of 40, and 8 groups of 9, then adding them.

Computation of 8x549: Ways to record general methods



- The first method proceeds from left to right, and the others from right to left. In the third method, the digits representing new units are written below the line rather than above 549, thus keeping the digits of the products close to each other, e.g., the 7 from 8x9=72 is written diagonally to the left rather than above the 4 in 549.

- When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works.

3546
-<u>928</u>

<u>*Student explanation for this problem:*</u>

1. There are not enough ones to take 8 ones from 6 ones so I have to use one ten as 10 ones. Now I have 3 tens and 16 ones. (Marks through the 4 and notates with a 3 above the 4 and writes a 1 above the ones column to be represented as 16 ones.

2. Sixteen ones minus 8 ones is 8 ones. (Writes an 8 in the ones column of answer.)

3. Three tens minus 2 tens is one ten. (Writes a 1 in the tens column of answer.)

4. There are not enough hundreds to take 9 hundreds from 5 hundreds so I have to use one thousand as 10 hundreds. (Marks through the 3 and notates with a 2 above it. (Writes down a 1 above the hundreds column.) Now I have 2 thousand and 15 hundreds.

5. Fifteen hundreds minus 9 hundreds is 6 hundreds. (Writes a 6 in the hundreds column of the answer).

6. I have 2 thousands left since I did not have to take away any thousands. (Writes 2 in the thousands place of answer.)

- Students should know that it is mathematically possible to subtract a larger number from a smaller number but that their work with whole numbers does not allow this as the difference would result in a negative number.

Common multiplication and division situations. [1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	**3 x ? = 18**, and **18 ÷ 3 = ?**	**? x 6 = 18**, and **18 ÷ 6 = ?**
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS [2], AREA [3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	**a x b = ?**	**a x ? = p** and **p ÷ a = ?**	**? x b = p**, and **p ÷ b = ?**

[1] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Module 1 Assessment / Authentic Assessment Recommended Framework			
Assessment	**CCSS**	**Estimated Time**	**Format**
Diagnostic Assessment (IREADY)		1-2 blocks	Individual
Eureka Math Module 1: Place Value, Round, and Algorithms for Addition/ Subtraction			
Authentic Assessment #1	4.NBT.4	30 mins	Individual
Optional Mid Module Assessment	4.OA.3 4.NBT.1-4	1 Block	Individual
Optional End of Module Assessment	4.OA.3 4.NBT.1-4	1 Block	Individual

Fourth Grade Ideal Math Block

Fluency: Whole Group
Sprints, Counting, Whiteboard Exchange

Application Problem: Whole Group
Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA

Concept Development: Individual/partner/whole
Instruction & Strategic Problem Set Questions

Student Debrief: Whole Group
Exit Ticket: Independent

50-60 min.

CENTERS/STATIONS:
Pairs / Small Group/ Individual
DIFFERENTIATED activities designed to **RETEACH, REMEDIATE, ENRICH** student's understanding of concepts.

20-30 min.

M: Meet with the teacher	**A: Application/ Problem Solving**	**T: Technology**	**H: Hands on Activities**

Eureka Lesson Structure:

Fluency:

- Sprints
- Counting : Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models
- Whiteboard Exchange

Application Problem:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

Concept Development: (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

PARCC Assessment Evidence/Clarification Statements			
CCSS	**Evidence Statement**	**Clarification**	**MP**
4.NBT.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 70 10 ÷ = by applying concepts of place value and division.	• None	MP.8
4.NBT.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using > =, , and	• Tasks assess conceptual understanding, e.g. by including a mixture (both within and between items) of expanded form, number names, and base ten numerals.	MP.7
4.NBT.4.1	Fluently add multi-digit whole numbers using the standard algorithm.	The given addends are such as to require an efficient/standard algorithm (e.g., 7263 + 4875). Addends in the task do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as 16,999 + 3,501). • Tasks do not have a context. • Grade 4 expectations in NJSLS are limited to whole numbers less than or equal to 1,000,000; for purposes of assessment, both of the given numbers should have 4 digits.	MP.7
4.NBT.4.2	Fluently subtract multi-digit whole numbers using the standard algorithm.	The given subtrahend and minuend are such as to require an efficient/standard algorithm (e.g. 7263 4875 − or 7406 4637). − The subtrahend and minuend do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as 7300 6301. −). • Tasks do not have a context. • Grade 4 expectations in NJSLS are limited to whole numbers less than or equal to 1,000,000; for purposes of assessment, both of the given numbers should have 4 digits.	MP.7

Number Talks

What does Number Talks look like?

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

Mental Math

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

Thumbs Up

- This is just a signal to let you know that you have given your students enough time to think about the problem
- If will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

Teacher as Recorder

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

Purposeful Problems

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

Starting Number Talks in your Classroom

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

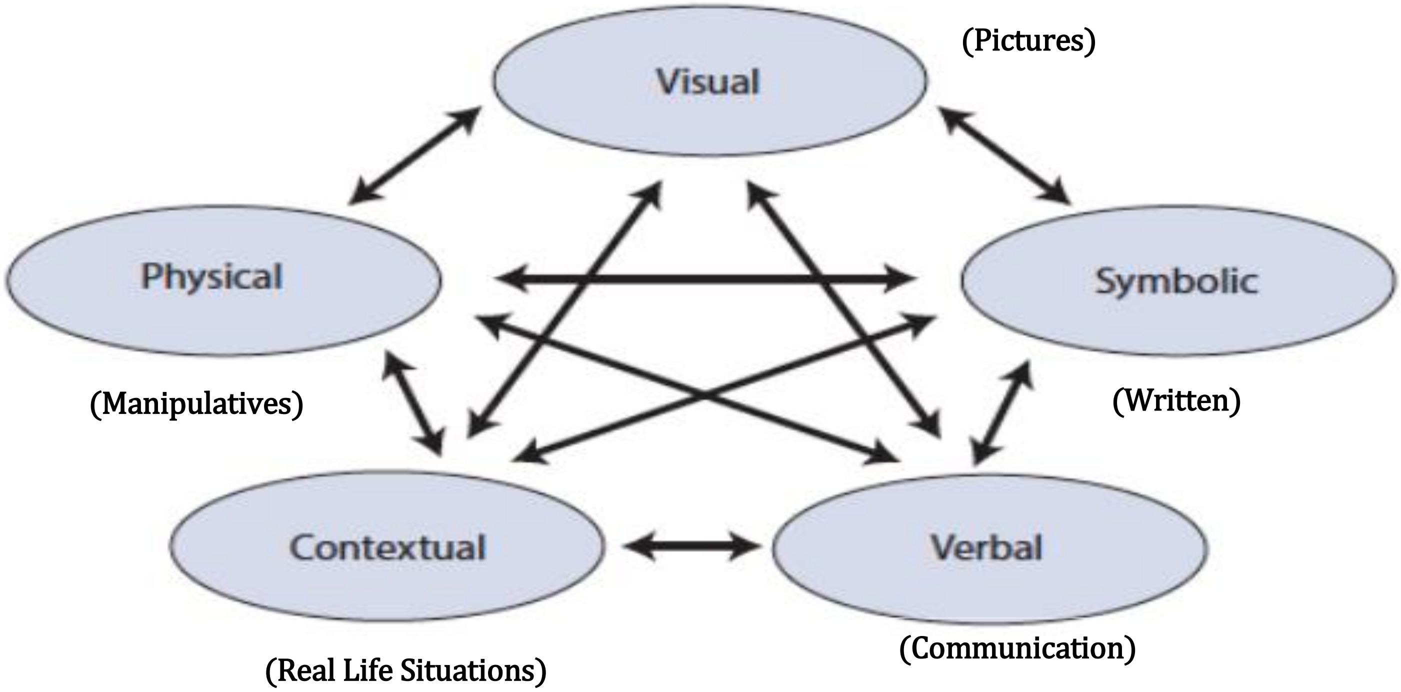
The teacher asks questions:

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?

Student Name: ______________________________ Task: _____________________ School: ________ Teacher: __________ Date: ________

"I CAN….."	STUDENT FRIENDLY RUBRIC				SCORE
	…a start 1	…getting there 2	…that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a class-mate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my think-ing.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: “Doing Stage”: Physical manipulation of objects to solve math problems.
Pictorial: “Seeing Stage”: Use of imaged to represent objects when solving math problems.
Abstract: “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.
DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?
WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

__Teacher Questioning:__
Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

100 questions that promote Mathematical Discourse

Help students **work together** to make sense of mathematics

1. What **strategy** did you use?
2. Do you **agree**?
3. Do you **disagree**?
4. Would you **ask the rest of the class** that question?
5. Could you **share your method** with the class?
6. What part of what he said **do you understand**?
7. Would someone like to **share** ___?
8. Can you **convince the rest of us** that your answer makes sense?
9. **What do others think** about what [student] said?
10. Can someone **retell or restate** [student]'s explanation?
11. Did you **work together**? In what way?
12. Would anyone like to **add to what was said**?
13. Have you **discussed** this with your group? With others?
14. Did anyone get a **different answer**?
15. **Where** would you go for **help**?
16. **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
17. How could you help another student **without telling them the answer**?
18. **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

19. Is this a **reasonable answer**?
20. Does that make **sense**?
21. **Why** do you think that? Why is that true?
22. Can you **draw a picture or make a model** to show that?
23. **How** did you reach that conclusion?
24. Does anyone want to **revise** his or her answer?
25. **How were you sure** your answer was right?

Help students learn to **reason mathematically**

26 **How did you begin** to think about this problem?

27 What is **another way** you could solve this problem?

28 How could you **prove** ______?

29 Can you **explain how your answer is different from or the same as** [student]'s answer?

30 Let's **break the problem into parts**. What would the parts be?

31 Can you **explain this part more specifically**?

32 Does that **always work**?

33 Can you think of a case where that **wouldn't work**?

34 How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

35 What do you need to do **next**?

36 What have you **accomplished**?

37 What are your **strengths and weaknesses**?

38 Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

39 What is this problem about? What can you **tell me about it**?

40 Do you need to **define or set limits** for the problem?

41 How would you **interpret** that?

42 Could you **reword that in simpler terms**?

43 Is there something that can be **eliminated** or that is **missing**?

44 Could you **explain** what the problem is asking?

45 What **assumptions** do you have to make?

46 What do you **know** about this part?

47 Which words were **most important**? Why?

Help students learn to **conjecture, invent, and solve** problems

48 **What would happen if ___?**

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

54 How is your solution method the **same as or different from** [student]'s method?

55 Other than retracing your steps, **how can you determine** if your answers are appropriate?

56 How did you **organize** the information? Do you have a **record**?

57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?

58 What have you tried? What **steps** did you take?

59 How would it look if you used this **model** or these **materials**?

60 How would you draw a **diagram or make a sketch** to solve the problem?

61 Is there **another possible answer**? If so, explain.

62 Is there **another way to solve** the problem?

63 Is there **another model** you could use to solve the problem?

64 Is there anything you've **overlooked**?

65 **How did you think** about the problem?

66 What was your **estimate or prediction**?

67 How **confident** are you in your answer?

68 **What else** would you like to know?

69 What do you think comes **next**?

70 Is the solution **reasonable**, considering the context?

71 Did you have a **system**? Explain it.

72 Did you have a **strategy**? Explain it.

73 Did you have a **design**? Explain it.

Help students learn to **connect mathematics, its ideas, and its application**

74 What is the **relationship** between ___ and ___?

75 Have we ever solved a problem **like this before**?

76 What uses of mathematics did you find in the **newspaper** last night?

77 What is the **same**?

78 What is **different**?

79 Did you use skills or build on concepts that were **not necessarily mathematical**?

80 Which **skills or concepts** did you use?

81 What **ideas** have we explored before that were useful in solving this problem?

82 Is there a **pattern**?

83 **Where else** would this strategy be useful?

84 How does this **relate** to ___?

85 Is there a **general rule**?

86 Is there a **real-life situation** where this could be used?

87 How would your method work with **other problems**?

88 What other problem does this seem to **lead to**?

Help students **persevere**

89 Have you tried making a **guess**?

90 **What else** have you tried?

91 Would **another method** work as well or better?

92 Is there **another way** to draw, explain, or say that?

93 Give me another **related problem**. Is there an easier problem?

94 How would you **explain** what you know right now?

Help students focus on the mathematics from **activities**

95 What was **one thing you learned** (or two, or more)?

96 Did you **notice any patterns**? If so, describe them.

97 What **mathematics topics** were used in this investigation?

98 What were the **mathematical ideas** in this problem?

99 What is mathematically **different about these two situations**?

100 What are the **variables** in this problem? What stays **constant**?

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

3-5 Math Fact Fluency Expectation

3.OA.C.7: Single-digit products and quotients (Products from memory by end of Grade 3)
3.NBT.A.2: Add/subtract within 1000
4.NBT.B.4: Add/subtract within 1,000,000/ Use of Standard Algorithm
5.NBT.B.5: Multi-digit multiplication/ Use of Standard Algorithm

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

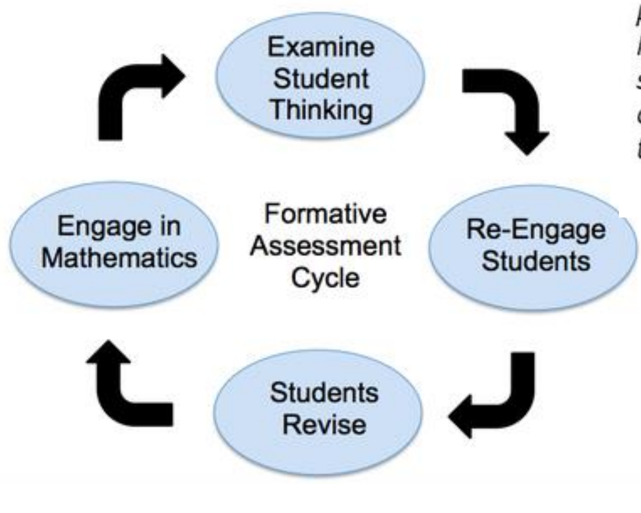
Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.



Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The Standards for Mathematical Practice:

Describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1	**Make sense of problems and persevere in solving them** Mathematically proficient students in grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
2	**Reason abstractly and quantitatively** Mathematically proficient fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3	**Construct viable arguments and critique the reasoning of others** In fourth grade mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4	**Model with mathematics** Mathematically proficient fourth grade students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evalu-

	ate their results in the context of the situation and reflect on whether the results make sense.
5	**Use appropriate tools strategically** Mathematically proficient fourth graders consider the available tools(including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
6	**Attend to precision** As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
7	**Look for and make use of structure** In fourth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
8	**Look for and express regularity in repeated reasoning** Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.

In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.
In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

Math Workstation: ____ Time: ___________

NJSLS.:

____
____
____
____

Objective(s): By the end of this task, I will be able to:

- ____
- ____
- ____

Task(s):

- ____
- ____
- ____
- ____

Exit Ticket:

- ____
- ____
- ____

MATH WORKSTATION SCHEDULE

Week of: ________________

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	BASED ON CURRENT OBSERVATIONAL DATA
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	

INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

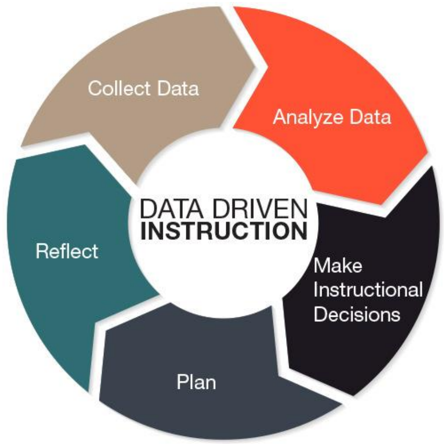
Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.



Data Analysis Form School: ________________ Teacher: ________________________ Date: _____________

Assessment: ____ NJSLS: ____

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

\

MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual", "Partner/Group", and "Individual w/Opportunity for Student Interviews[1].
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended.*
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)[2].
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

4th Grade Authentic Assessment #1 – Zoo Animals

Name:_______________________________________

The animals at the zoo were weighed.
Name 3 animal pairs that have a difference in weight that is greater than 2,000.
<u>Show all work</u> including the actual differences for the 3 pairs of animals.

Animal	Weight in Pounds
Giraffe	2,685
Polar Bear	685
Hippopotamus	3,086
Cheetah	144
Buffalo	1600

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.
Mathematical Practice: 1, 3, 6
Type: Individual

SOLUTION: **Pair 1: Giraffe & Cheetah (2,541 pounds)** **Pair 2: Hippopotamus & Cheetah (2,942 pounds)** **Pair 3: Hippopotamus & Polar Bear (2,401 pounds)**				
Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Accurately and **quickly** adds and subtracts multi-digit whole numbers using the standard algorithm. Response includes an **efficient** and logical progression of steps.	Accurately and **in a timely manner** adds or subtracts multi-digit whole numbers using the standard algorithm. Response includes a **logical** progression of steps	**Accurately** adds and subtracts multi-digit whole numbers using the standard algorithm. Response includes a **logical but incomplete** progression of steps. Minor calculation errors.	Adds and subtracts multi-digit whole numbers using the standard algorithm with some level of accuracy. Response includes an **incomplete or Illogical** progression of steps.	Does not address task, unresponsive, unrelated or inappropriate.

Resources

Great Minds
https://greatminds.org/

Embarc
https://embarc.online/

Engage NY
http://www.engageny.org/video-library?f[0]=im_field_subject%3A19

Common Core Tools
http://commoncoretools.me/
http://www.ccsstoolbox.com/
http://www.achievethecore.org/steal-these-tools

Achieve the Core
http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12

Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html

http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000

http://www.thinkingblocks.com/

Illustrative Math Project :http://illustrativemathematics.org/standards/k8

Inside Mathematics: http://www.insidemathematics.org/index.php/tools-for-teachers

Sample Balance Math Tasks: http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/

Georgia Department of Education:https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx

Gates Foundations Tasks:http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf

Minnesota STEM Teachers' Center:
http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships

Singapore Math Tests K-12: http://www.misskoh.com

Mobymax.com: http://www.mobymax.com

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

References

"Eureka Math" *Great Minds.* 2018 < https://greatminds.org/account/products>